Mechanical System

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Ms. V. ANBUVALLI
Assistant Professor
Department of Mathematics
Shrimati Indira Gandhi College
Trichy.

Mechanical System

Mechanics:

Mechanics is a science that deals with the action of forces on the bodies. It can be classified into to two branches.

i) Dynamics

ii) Statics

- Dynamics deals with the bodies having motion
- Statics deals with the bodies at rest.

Classical Dynamics:

Classical Dynamics is restricted to those system of interacting bodies for which quantum mechanical effects are negligible that is it applies primarily to Macroscopic phenomena.

Force:

A vector quantity that describes an interaction that, when unopposed, will change the motion of an object. It can cause an object with mass to accelerate.

In Newton's second law, force is directly proportional to the mass and acceleration of an object: $F = ma$.

Mechanical System:

A collection of physical objects whose motion is governed by the principles of classical mechanics and that interact through forces and constraints.

Displacement (r or Δr):

A vector quantity that refers to the change in position of an object. It has both magnitude (the distance between the initial and final points) and direction.

Velocity (v):

A vector quantity that describes the rate of change of an object's position with respect to time. It also has both magnitude (speed) and direction. Mathematically, it's the first derivative of the position vector with respect to time: v=dt dr.

Acceleration (a):

A vector quantity that describes the rate of change of an object's velocity with respect to time. It also has both magnitude and direction. Mathematically, it's the first derivative of the velocity vector (or the second derivative of the position vector) with respect to time:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

Linear Momentum (p):

A vector quantity defined as the product of an object's mass (m) and its velocity (v): p=mv. It represents the "quantity of motion" of an object. Newton's second law can also be expressed in terms of momentum: the net force on an object is equal to the rate of change of its linear momentum: F=dt dp.

Angular Momentum (L):

A vector quantity that measures the "amount of rotation" of an object with respect to a chosen axis or point. For a particle with linear momentum p at a position vector r relative to the axis/point, the angular momentum is given by the cross product: L= r×p. For a rigid body, it involves the moment of inertia and angular velocity. The net torque on an object is equal to the rate of change of its angular momentum: τ=dtdL

Mechanical System:

A collection of physical objects (particles, rigid bodies, etc.) whose motion is governed by the laws of classical mechanics and which interact through forces.

Degrees of Freedom:

The minimum number of independent coordinates required to completely specify the configuration of a mechanical system at any instant of time.

ie) The number of degree of freedom is equal to the number of coordinates minus number of force independent equations of constraints.

Generalized Coordinates (qi**):**

A set of independent coordinates that fully describe the configuration of a mechanical system. Their number is equal to the degrees of freedom of the system. These coordinates do not necessarily have to be Cartesian coordinates.

Configuration Space:

An n-dimensional space where each point represents a unique configuration of a system with n degrees of freedom. The generalized coordinates serve as the axes of this space.

Constraints:

A system of n particles may have less than 3N degree of freedom because presence of the constraints.

Classification of Constraints:

- **Holonomic Constraints:** Constraints that can be expressed as equations relating the generalized coordinates and possibly time: f $(q_1, q_2, \ldots, q_n, t) = 0$. These constraints reduce the number of independent generalized coordinates.

- **Nonholonomic Constraints:** Constraints that cannot be expressed in the above form, often involving velocities or inequalities. An example is a rolling without slipping condition.

- **Scleronomic Constraints:** Constraints that do not explicitly depend on time.

- **Rheonomic Constraints:** Constraints that explicitly depend on time.
- **Bilateral Constraints:** Constraints that can be expressed as equalities, allowing motion in both directions along the constraint.
- **Unilateral Constraints:** Constraints that can be expressed as inequalities, restricting motion to one side of a boundary (e.g., an object resting on a surface).

Virtual Displacement (δri **or** δqi**):**

An infinitesimal, instantaneous change in the configuration of a system that is consistent with the constraints at a given instant in time. Virtual displacements are hypothetical and do not necessarily correspond to an actual motion of the system.

Virtual Work (δW**):**

The work done by the forces in a virtual displacement is called a virtual work done by the forces.

Principle of Virtual work:

The necessary and sufficient condition for the static equilibrium of an initially motionless scleronomic system, which is subject to workless constraints is zero is that the virtual work done by the applied force in moving through a virtual displacement satisfying the constraints is zero.

Generalized Force (Q_j):

A quantity associated with each generalized coordinate q_j such that the virtual work done by all the active forces during a virtual displacement δq_j is given by $\delta W = \sum Q_j \delta q_j$. The generalized forces account for both conservative and non-conservative forces.

Kinetic Energy (T):

The energy of motion of a system, expressed in terms of the generalized coordinates and their time derivatives (generalized velocities, $q_{\cdot i}$).

Potential Energy (V):

The energy associated with the configuration of a system and the conservative forces acting on it. The force is related to the potential energy by $F=-\nabla V$.

Equilibrium:

An equilibrium configuration of a conservative holonomic system with workless fixied constraints must occur at a point where the potential energy has a stationary value.(ie) $\frac{dv}{dq_i} = 0$

Stability:

A position of equilibrium for any system is said to be stable when an arbitrary small disturbance does not cause the system depart for from the position of equilibrium otherwise it is called to be unstable.

Lagrangian (L):

A function that describes the dynamics of a system, defined as the difference between the kinetic energy and the potential energy: $L = T - V$.

Hamiltonian (H):

Another function that describes the dynamics of a system, often used in more advanced formulations. It is related to the Lagrangian through a Legendre transformation: $H = \sum p_i \dot{q}_i - L$, where p_i are the generalized momenta. For conservative systems, the Hamiltonian often represents the total energy of the system.

- ***THANK YOU***